AME 486

Project 7

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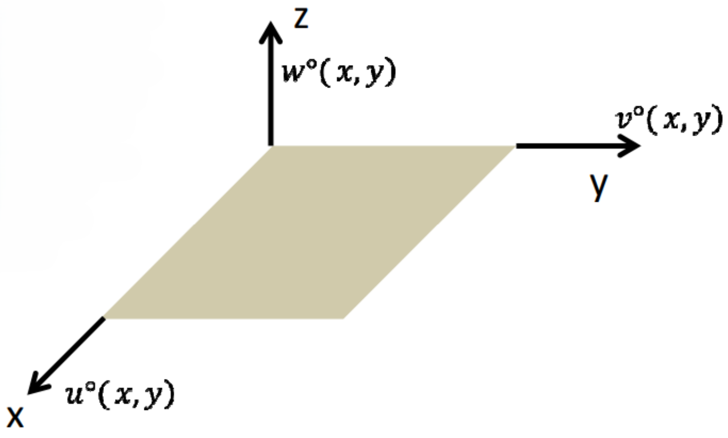
#1

- Composite plate theory
    - Where a 2D structural representation is used as an idealization and simplification of a 3D thin or thick laminate
    - Only use the midplane reference of the plate to characterize the entire system
        - Use the mid surface displacements
        - Strains
        - Stresses
            - Equilibrium and Boundary conditions
            - Force and Moment Resultants
        - Constitutive Law
        - Summary of equations
    - Step 1 - Deflections are first tracked for the mid-surface
        - Z = 0 – because mid-surface measurements only



deflection

(x) $U(x,y,z) = U^\circ(x,y) - z\,\cancel{\frac{dw^\circ(x,y)}{dx}}$

(y) $V(x,y,z) = V^\circ(x,y) - z\,\cancel{\frac{dw^\circ(x,y)}{dy}}$

(z) $w = (x,y,z) = w^\circ(x,y)$

$z = 0$

#1

- Transverse shear strains are zero, normal strains remain the same after deformation

$$2\varepsilon_{xz} = \frac{du}{dz} + \frac{dw}{dx} = -\frac{dw^0}{dx} + \frac{dw^0}{dx} = 0$$

$$2\varepsilon_{yz} = \frac{dv}{dz} + \frac{dw}{dy} = -\frac{dw^0}{dy} + \frac{dw^0}{dy} = 0$$

- There is no Z-direction (thickness) changes therefore, transverse normal strain is zero

$$\varepsilon_z = \frac{dw}{dz} = \frac{dw^0}{dz} = 0$$

#1

- Using the strain-displacement relationship strains are calculated
  - This is with respect to the thickness
  - Curvatures are added

$$\underbrace{\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix}}_{\text{Midsurface}} + z \underbrace{\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}}_{\text{Curvatures}}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \partial u^0/\partial x \\ \partial v^0/\partial y \\ \dfrac{\partial u^0}{\partial y} + \dfrac{\partial v^0}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} -\partial^2 w^0/\partial x^2 \\ -\partial^2 w^0/\partial y^2 \\ -2\,\partial^2 w^0/\partial x \partial y \end{Bmatrix}$$

#1

- For mid-surface planes, the governing equations are different than in 3D elasticity because we are not characterizing a 3D plate. Thickness is zero. Governing equations for a 2D plate need to be derived
- A 2D plate simplifies this process because we only consider the mid-surface displacements as functions of x and y
  - Governing equations for the 2D plate align with the assumed displacement field
  - Stress quantities are introduced to represent internal forces acting on the mid-surface of the plate
- A lengthy integration takes place, and the fundamental governing equations is as shown
  - There are 2 stress equations and 1 bending equation

$$\frac{dN_x}{dx} + \frac{dN_{xy}}{dy} = 0$$

$$\frac{dN_{xy}}{dx} + \frac{dN_y}{dy} = 0$$

$$\frac{d^2 M_x}{dx^2} + 2\frac{d^2 M_{xy}}{dx\,dy} + \frac{d^2 M_y}{dy^2} + p(x,y) = 0$$

#1

- There are 6 resultants
- Force (N) and moment (M) are found by integrating the 3D stress field through the thickness
  - 3 from the equation involving force resultants
  - 3 from the equation involving moment resultants

- The plate has multiple layers so the integral needs to be re-written
  - Remember, every layer is different angles
  - Note: the moment resultant is for the global coordinates – we still need to find the local

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} dz \quad \longrightarrow \quad \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} dz$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} z \, dz \qquad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} z \, dz$$

#1

- In plate theory, the out of plane stresses are near zero since the plate is thin (mid-surface)
    - Stresses do not develop significantly over the thickness

$$\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$$

$$\sigma_{xx},\ \sigma_{yy},\ \tau_{xy} \neq 0$$

- The Constitutive law is applied to find the strain-stress relationship for plane stress
    - This is found from inversion
    - Only need to know four properties (E_1, E_2, v_{12}, G_{12})

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{v_{21}}{E_{22}} & 0 \\ \frac{-v_{12}}{E_{11}} & \frac{1}{E_{22}} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} \longrightarrow \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{Bmatrix}$$

$$[Q] = [S]^{-1}$$

$$Q_{11} = \frac{E_1}{1 - v_{12}v_{21}} \qquad Q_{22} = \frac{E_2}{1 - v_{12}v_{21}}$$

$$Q_{12} = \frac{v_{12}E_2}{1 - v_{12}v_{21}} \qquad Q_{66} = G_{12}$$

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- Each layer is characterized and transformed to a laminate coordinate system
  - $m = \cos\alpha$
  - $n = \sin\alpha$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \qquad \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} = \gamma_{12}/2 \end{Bmatrix} = [T] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} = \gamma_{xy}/2 \end{Bmatrix}$$

- Short notation

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} [Q] [T] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}$$

#1

- Matrix multiplication is done to convert to engineering strains
  - We get the final form

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

  - Using trig identities, stiffness quantities can be solved for as a function of ply angles and invariant quantities *U*

$$\bar{Q}_{11} = U_1 + U_2 \cos 2\theta + U_3 \cos 4\theta$$

$$\bar{Q}_{12} = U_4 - U_3 \cos 4\theta$$

$$\bar{Q}_{22} = U_1 - U_2 \cos 2\theta + U_3 \cos 4\theta$$

$$\bar{Q}_{16} = \frac{U_2}{2} \sin 2\theta + U_3 \sin 4\theta$$

$$\bar{Q}_{26} = \frac{U_2}{2} \sin 2\theta - U_3 \sin 4\theta$$

$$U_1 = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})$$

$$U_2 = \frac{1}{2}(Q_{11} - Q_{22})$$

$$U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})$$

$$U_4 = \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})$$

$$U_5 = \frac{1}{8}(Q_{11} + Q_{22} + 4Q_{66} - 2Q_{12})$$

#1

- Using the force resultant and constitutive law we get a strain relationship

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}^{(k)} \left[(z_k - z_{k-1}) \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \frac{1}{2}(z_k^2 - z_{k-1}^2) \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \right]$$

- Then rewritten as the following
  - N = In-plane internal force resultants
  - A = Extensional stiffness
    - A_{16} and A_{26} = Shear-extension coupling
  - $\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0$ = In-plane strains at mid-surface
  - B = Bending-extensional coupling
  - K = Curvatures

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

$$A_{ij} = \sum_{k=1}^{N} \bar{Q}_{ij}^{(k)} (z_k - z_{k-1}) \qquad B_{ij} = \frac{1}{2} \sum_{k=1}^{N} \bar{Q}_{ij}^{(k)} (z_k^2 - z_{k-1}^2)$$

#1

- Using the moment resultant and constitutive law we get a strain relationship as well

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}^{(k)} \left[\frac{1}{2}\left(z_k^2 - z_{k-1}^2\right) \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \frac{1}{3}\left(z_k^3 - z_{k-1}^3\right) \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \right]$$

- Then rewritten as the following
  - M = Moment resultants
  - B = bending-extensional coupling
  - $\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0$ = In-plane strains at mid-surface
  - D = Bending stiffness
    - D_{16} and D_{26} = Bending-twist coupling
  - K = Curvatures

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

$$B_{ij} = \frac{1}{2} \sum_{k-1}^{N} \bar{Q}_{ij}^{(k)} \left(z_k^2 - z_{k-1}^2\right) \qquad D_{ij} = \frac{1}{3} \sum_{k-1}^{N} \bar{Q}_{ij}^{(k)} \left(z_k^3 - z_{k-1}^3\right)$$

#1

- The final form of the ABD matrix is as follows

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

$$A_{ij} = \sum_{k-1}^{N} \bar{Q}_{ij}^{(k)} \left(z_k - z_{k-1} \right) \qquad B_{ij} = \frac{1}{2} \sum_{k-1}^{N} \bar{Q}_{ij}^{(k)} \left(z_k^2 - z_{k-1}^2 \right)$$

$$D_{ij} = \frac{1}{3} \sum_{k-1}^{N} \bar{Q}_{ij}^{(k)} \left(z_k^3 - z_{k-1}^3 \right)$$

- Short-hand notion is

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j^0 \\ \kappa_j \end{Bmatrix}$$

- Where,
  - N = Force Resultant
  - M = Moment resultant
  - A_{ij} = Stretching plate stiffness
  - B_{ij} = Coupling plate stiffness
  - D_{ij} = Bending plate stiffness
- i, j = x, y, xy
- i, j = 1,2,6

#1

- Short-hand notion is

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j^0 \\ \kappa_j \end{Bmatrix}$$

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  - D_{ij} = Bending plate stiffness
- i, j = x, y, xy
- i, j = 1,2,6

- Conclusion
  - FEA uses these complex equations and discretizes larger elements into smaller elements
  - With a refined mesh, you are able to get detailed stress info for each ply
  - For course element models, it is better to extract loads N and M for failure analysis
  - The closed form solutions are good to have
    - You can use equations to calculate stress in each ply
    - Its quick
    - Its cheap
    - Sanity check and estimation of what you should expect

#2

- Assuming an AS4/3501-6 composite unidirectional material system. The following properties were based on: https://www.researchgate.net/publication/225141453_Influence_of_the_Matrix_Type_on_the_Mode_I_Fracture_of_Carbon-Epoxy_Composites_Under_Dynamic_Delamination
  - E11 = 131 GPa = 1.31e+11 Pa
  - E22 = 8.9 GPa = 8.9e+9 Pa
  - G12 = 5.09 GPa = 5.09e+9 Pa
  - V12 = 0.3
  - Alpha_1 = 1.954e+09 Pa
  - Alpha _2 = 2.4e+07 Pa

  - The loading conditions for our example would be
    - [N] = [N_x = 1000, N_y = 100, N_xy = 100
    - [M] = [M_x = 50, M_y = 25, M_xy = 25]

#2 (a)

Using the Provided Excel Sheet for [0/-45/45/90]s

- Material properties are input into the code
  - Thickness, angle, and ply # are inputted as well

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	A	B	C	D	E	F
1	**Laminate**					
2						
3	Enter layer information in bold coloumns from top (layer 1) to bottom (max 30)					
4						
5	LAYER #	**Mat. ID**	Material Name	**Thickness**	**Angle (°)**	
6	1	**6**	AS4/3501-6	**0.000125**	**0**	
7	2	**6**	AS4/3501-6	**0.000125**	**-45**	
8	3	**6**	AS4/3501-6	**0.000125**	**45**	
9	4	**6**	AS4/3501-6	**0.000125**	**90**	
10	5	**6**	AS4/3501-6	**0.000125**	**90**	
11	6	**6**	AS4/3501-6	**0.000125**	**45**	
12	7	**6**	AS4/3501-6	**0.000125**	**-45**	
13	8	**6**	AS4/3501-6	**0.000125**	**0**	
14	9					
15	10					
16	11					
17	12					
18	13					
19	14					

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	A	B	C	D	E	F	G
1	**Materials**						
2							
3		in Pa	in Pa	in Pa	in Pa	in Pa	Can be Pa or psi
4	Material Number	**1**	**2**	**3**	**4**	**5**	**6**
5	Material Name	CFRP	E-Glass	A36 Steel	6061 Aluminum	PVC Foam	AS4/3501-6
6	E_11	1.500E+11	7.000E+10	2.000E+11	7.000E+10	5.000E+08	1.310E+11
7	E_22	1.000E+10	1.000E+10	2.000E+11	7.000E+10	5.000E+08	8.900E+09
8	v_12	0.25	0.25	0.32	0.35	0.45	0.3
9	G_12	8.000E+09	5.000E+09	7.570E+10	2.590E+10	1.724E+08	5.090E+09
10	alpha_1	-1.000E-06	4.000E-06	6.600E-06	1.310E-05	4.000E-05	1.954E+09
11	alpha_2	1.000E-05	1.250E-05	6.600E-06	1.310E-05	4.000E-05	2.400E+07
12							

Using the Provided Excel Sheet for [0/-45/45/90]s

#2 (a)

- Made up loading conditions are added
  - This remained consistent in both Part (a) and Part (b)
  - There is no temperature gradient / no thermal effects occurring

	A	B	C
1	**Loading**		
2			
3	Membrane loads and bending moments:		
4	N_x	1000	
5	N_y	100	
6	N_xy	100	
7	M_x	50	
8	M_y	25	
9	M_xy	25	
10			
11	Temperature gradient from cure:		
12	Delta T	0	

Using the Provided Excel Sheet for [0/-45/45/90]s

#2 (a)

ABD Matrix:

56001862.19	17064904.84	0	9.09495E-13	2.55795E-13	0.0000E+00
17064904.84	56001862.19	1.86265E-09	2.55795E-13	-6.25278E-13	0
0	1.86265E-09	19468478.67	0	0	0
9.09495E-13	2.55795E-13	0	7.770809968	1.197411674	-0.479887398
2.55795E-13	-6.25278E-13	0	1.197411674	2.012161188	-0.479887398
0.0000E+00	0	0	-0.479887398	-0.479887398	1.397709494

Inverted ABD Matrix:

1.96843E-08	-5.99821E-09	5.73879E-25	-1.63007E-21	-3.84455E-21	-1.87965E-21
-5.99821E-09	1.96843E-08	-1.8833E-24	-1.04106E-21	8.07489E-21	2.41498E-21
5.73879E-25	-1.8833E-24	5.13651E-08	9.96031E-38	-7.72564E-37	-2.31053E-37
-1.63007E-21	-1.04106E-21	9.96031E-38	0.142271342	-0.07952604	0.021542874
-3.84455E-21	8.07489E-21	-7.72564E-37	-0.07952604	0.585755073	0.173807887
-1.87965E-21	2.41498E-21	-2.31053E-37	0.021542874	0.173807887	0.782527681

	Applied Loading		Thermal Loading		Total Loading		Strains/Curvatures
N_x	1000	N_xT	0.000E+00		1.000E+03		1.908E-05
N_y	100	N_yT	0.000E+00		1.000E+02		-4.030E-06
N_xy	100	N_xyT	0.000E+00	=	1.000E+02	Gives	5.137E-06
M_x	50	M_x	0.000E+00		5.000E+01		5.664E+00
M_y	25	M_y	0.000E+00		2.500E+01		1.501E+01
M_xy	25	M_xy	0.000E+00		2.500E+01		2.499E+01

#2 (a)

Using the Our Own Excel Sheet for [0/-45/45/90]s

- Compared to our own Excel sheet
  - The answers are exactly the same
  - We used the given excel code to make our own
    - The process was not hard but gave insight into how it worked
  - We changed the colors of the matrix to distinguish our excel code from the one given in class
    - The formulas were exactly the same – hence, there is only one correct answer

ABD

56001862.19	17064904.84	0	9.09495E-13	2.55795E-13	0.0000E+00
17064904.84	56001862.19	1.86265E-09	2.55795E-13	-6.25278E-13	0
0	1.86265E-09	19468478.67	0	0	0
9.09495E-13	2.55795E-13	0	7.770809968	1.197411674	-0.479887398
2.55795E-13	-6.25278E-13	0	1.197411674	2.012161188	-0.479887398
0.0000E+00	0	0	-0.479887398	-0.479887398	1.397709494

ABD Inverted

1.96843E-08	-5.99821E-09	5.73879E-25	-1.63007E-21	-3.84455E-21	-1.87965E-21
-5.99821E-09	1.96843E-08	-1.8833E-24	-1.04106E-21	8.07489E-21	2.41498E-21
5.73879E-25	-1.8833E-24	5.13651E-08	9.96031E-38	-7.72564E-37	-2.31053E-37
-1.63007E-21	-1.04106E-21	9.96031E-38	0.142271342	-0.07952604	0.021542874
-3.84455E-21	8.07489E-21	-7.72564E-37	-0.07952604	0.585755073	0.173807887
-1.87965E-21	2.41498E-21	-2.31053E-37	0.021542874	0.173807887	0.782527681

Strains
1.908E-05
-4.030E-06
5.137E-06
5.664E+00
1.501E+01
2.499E+01

Using the Provided Excel Sheet for [0/-45/45/90]s

#2 (a)

Top of the Layer

Mechanical Strains and Stresses (Locally Layer by Layer)								
eps_1	2.851E-03	-8.029E-04	5.718E-03	1.873E-03	-4.030E-06	-2.844E-03	5.436E-04	-2.105E-03
eps_2	7.502E-03	8.572E-03	-5.336E-04	7.271E-04	1.908E-05	2.743E-04	-5.698E-03	-5.634E-03
gamma_12	1.250E-02	-3.483E-03	2.314E-03	-3.128E-03	-5.137E-06	-1.192E-03	2.360E-03	-9.364E-03
sig_1	3.959E+08	-8.280E+07	7.522E+08	2.488E+08	-4.799E+05	-3.741E+08	5.634E+07	-2.926E+08
sig_2	7.484E+07	7.460E+07	1.058E+07	1.154E+07	1.601E+05	-5.184E+06	-4.956E+07	-5.610E+07
tau_12	6.361E+07	-1.773E+07	1.178E+07	-1.592E+07	-2.614E+04	-6.066E+06	1.201E+07	-4.766E+07

Bottom of the Layer

Mechanical Strains and Stresses (Locally Layer by Layer)								
eps_1	2.143E-03	-5.336E-04	2.864E-03	-4.030E-06	-1.881E-03	-5.698E-03	8.129E-04	-2.813E-03
eps_2	5.626E-03	5.718E-03	-2.643E-04	1.908E-05	-6.889E-04	5.436E-04	-8.552E-03	-7.510E-03
gamma_12	9.375E-03	-2.314E-03	1.145E-03	-5.137E-06	3.118E-03	-2.360E-03	3.529E-03	-1.249E-02
sig_1	2.976E+08	-5.498E+07	3.768E+08	-4.799E+05	-2.497E+08	-7.495E+08	8.417E+07	-3.909E+08
sig_2	5.613E+07	4.977E+07	5.327E+06	1.601E+05	-1.122E+07	-1.044E+07	-7.439E+07	-7.481E+07
tau_12	4.772E+07	-1.178E+07	5.831E+06	-2.614E+04	1.587E+07	-1.201E+07	1.796E+07	-6.356E+07

Using Our Own Excel Sheet for [0/-45/45/90]s

#2 (a)

Top of the Layer – Values are the same

ε_x	2.851E-03	2.143E-03	1.435E-03	7.271E-04	1.908E-05	-6.889E-04	-1.397E-03	-2.105E-03
ε_y	7.502E-03	5.626E-03	3.749E-03	1.873E-03	-4.030E-06	-1.881E-03	-3.757E-03	-5.634E-03
γ_{xy}	1.250E-02	9.375E-03	6.252E-03	3.128E-03	5.137E-06	-3.118E-03	-6.241E-03	-9.364E-03
σ_x	3.959E+08	-8.280E+07	7.522E+08	2.488E+08	-4.799E+05	-3.741E+08	5.634E+07	-2.926E+08
σ_y	7.484E+07	7.460E+07	1.058E+07	1.154E+07	1.601E+05	-5.184E+06	-4.956E+07	-5.610E+07
τ_{xy}	6.361E+07	-1.773E+07	1.178E+07	-1.592E+07	-2.614E+04	-6.066E+06	1.201E+07	-4.766E+07

Bottom of the Layer – Values are the same

ε_x	2.143E-03	1.435E-03	7.271E-04	1.908E-05	-6.889E-04	-1.397E-03	-2.105E-03	-2.813E-03
ε_y	5.626E-03	3.749E-03	1.873E-03	-4.030E-06	-1.881E-03	-3.757E-03	-5.634E-03	-7.510E-03
γ_{xy}	9.375E-03	6.252E-03	3.128E-03	5.137E-06	-3.118E-03	-6.241E-03	-9.364E-03	-1.249E-02
σ_x	2.976E+08	-5.498E+07	3.768E+08	-4.799E+05	-2.497E+08	-7.495E+08	8.417E+07	-3.909E+08
σ_y	5.613E+07	4.977E+07	5.327E+06	1.601E+05	-1.122E+07	-1.044E+07	-7.439E+07	-7.481E+07
τ_{xy}	4.772E+07	-1.178E+07	5.831E+06	-2.614E+04	1.587E+07	-1.201E+07	1.796E+07	-6.356E+07

#2 (b)

Using the Provided Excel Sheet for [0/+30/-30/90]s

Laminate

Enter layer information in bold coloumns from top (layer 1) to bottom (max 30)

LAYER #	**Mat. ID**	Material Name	**Thickness**	**Angle (°)**
1	**6**	AS4/3501-6	**0.000125**	**0**
2	**6**	AS4/3501-6	**0.000125**	**30**
3	**6**	AS4/3501-6	**0.000125**	**-30**
4	**6**	AS4/3501-6	**0.000125**	**90**
5	**6**	AS4/3501-6	**0.000125**	**90**
6	**6**	AS4/3501-6	**0.000125**	**-30**
7	**6**	AS4/3501-6	**0.000125**	**30**
8	**6**	AS4/3501-6	**0.000125**	**0**
9				
10				
11				

Note :

- [N] and [M} were kept constant, and the properties of the composite were kept constant
- Our homemade excel sheet works - as shown in part (a). No need to compare our excel sheet to part (b) – the answers are the same

Using the Provided Excel Sheet for [0/+30/-30/90]s

#2 (b)

	74952878.6	13470285.17	0	0	2.84217E-14	0.0000E+00
	13470285.17	44240085.11	1.86265E-09	2.84217E-14	2.84217E-13	0
ABD Matrix:	0	1.86265E-09	15873859.01	0	0	0
	0	2.84217E-14	0	9.053951705	0.954025967	0.610159175
	2.84217E-14	2.84217E-13	0	0.954025967	1.215790865	0.221030181
	0.0000E+00	0	0	0.610159175	0.221030181	1.154323787

	1.4114E-08	-4.29746E-09	5.04266E-25	-5.69286E-23	7.39626E-22	-1.11532E-22
	-4.29746E-09	2.39124E-08	-2.80589E-24	5.0183E-22	-6.04559E-21	8.92351E-22
Inverted	5.04266E-25	-2.80589E-24	6.29967E-08	-5.88849E-38	7.09392E-37	-1.04709E-37
ABD Matrix:	-5.69286E-23	5.0183E-22	-5.88849E-38	0.122938503	-0.087708572	-0.048189091
	7.39626E-22	-6.04559E-21	7.09392E-37	-0.087708572	0.914749276	-0.128794892
	-1.11532E-22	8.92351E-22	-1.04709E-37	-0.048189091	-0.128794892	0.916441805

	Applied Loading		**Thermal Loading**		**Total Loading**		**Strains/Curvatures**
N_x	1000	N_xT	0.000E+00		1.000E+03		1.368E-05
N_y	100	N_yT	0.000E+00		1.000E+02		-1.906E-06
N_xy	100	N_xyT	0.000E+00	**=**	1.000E+02	**Gives**	6.300E-06
M_x	50	M_x	0.000E+00		5.000E+01		2.749E+00
M_y	25	M_y	0.000E+00		2.500E+01		1.526E+01
M_xy	25	M_xy	0.000E+00		2.500E+01		1.728E+01

#2 (b)

Using the Provided Excel Sheet for [0/+30/-30/90]s

Top of the Layer

Mechanical Strains and Stresses (Locally Layer by Layer)								
eps_1	1.388E-03	5.023E-03	-3.942E-04	1.906E-03	-1.906E-06	2.077E-04	-3.328E-03	-1.017E-03
eps_2	7.630E-03	1.744E-03	4.909E-03	3.574E-04	1.368E-05	-2.448E-03	-1.164E-03	-5.726E-03
gamma_12	8.647E-03	7.294E-03	-5.325E-04	-2.167E-03	-6.300E-06	2.912E-04	-4.880E-03	-6.474E-03
sig_1	2.035E+08	6.667E+08	-3.878E+07	2.522E+08	-2.145E+05	2.080E+07	-4.417E+08	-1.495E+08
sig_2	7.205E+07	2.911E+07	4.290E+07	8.321E+06	1.174E+05	-2.136E+07	-1.936E+07	-5.401E+07
tau_12	4.401E+07	3.713E+07	-2.710E+06	-1.103E+07	-3.207E+04	1.482E+06	-2.484E+07	-3.295E+07

Bottom of the Layer

Mechanical Strains and Stresses (Locally Layer by Layer)								
eps_1	1.045E-03	3.353E-03	-1.936E-04	-1.906E-06	-1.910E-03	4.084E-04	-4.998E-03	-1.361E-03
eps_2	5.722E-03	1.162E-03	2.457E-03	1.368E-05	-3.300E-04	-4.900E-03	-1.745E-03	-7.634E-03
gamma_12	6.487E-03	4.859E-03	-2.579E-04	-6.300E-06	2.154E-03	5.658E-04	-7.315E-03	-8.635E-03
sig_1	1.531E+08	4.450E+08	-1.892E+07	-2.145E+05	-2.526E+08	4.066E+07	-6.634E+08	-1.999E+08
sig_2	5.404E+07	1.941E+07	2.148E+07	1.174E+05	-8.086E+06	-4.278E+07	-2.905E+07	-7.201E+07
tau_12	3.302E+07	2.473E+07	-1.313E+06	-3.207E+04	1.096E+07	2.880E+06	-3.723E+07	-4.395E+07

Comparison between Both Cases

#2 (a)

[0/-45/45/90]s

	Applied Loading		Thermal Loading		Total Loading		Strains/Curvatures
N_x	1000	N_xT	0.000E+00		1.000E+03		1.908E-05
N_y	100	N_yT	0.000E+00		1.000E+02		-4.030E-06
N_xy	100	N_xyT	0.000E+00	=	1.000E+02	Gives	5.137E-06
M_x	50	M_x	0.000E+00		5.000E+01		5.664E+00
M_y	25	M_y	0.000E+00		2.500E+01		1.501E+01
M_xy	25	M_xy	0.000E+00		2.500E+01		2.499E+01

#2 (b)

[0/+30/-30/90]s

	Applied Loading		Thermal Loading		Total Loading		Strains/Curvatures
N_x	1000	N_xT	0.000E+00		1.000E+03		1.368E-05
N_y	100	N_yT	0.000E+00		1.000E+02		-1.906E-06
N_xy	100	N_xyT	0.000E+00	=	1.000E+02	Gives	6.300E-06
M_x	50	M_x	0.000E+00		5.000E+01		2.749E+00
M_y	25	M_y	0.000E+00		2.500E+01		1.526E+01
M_xy	25	M_xy	0.000E+00		2.500E+01		1.728E+01

From the both cases, we can see:

- The laminate in (a) has ± 45 plies, which contribute significantly to shear stiffness and 90 ° provided less stiffness in the axial direction
    - We can see there is higher axial strain (e_x) in part (a) than in (b), which makes sense as they are more compliant under axial loading compared to ± 30 plies
- It is also noticeable that the shear strain (gamma_xy) is lower in (a) than (b)
    - Since it is known that ± 30 plies provide more anisotropic shear response, we can expect to see a higher shear strain

Comparison between Both Cases

#2 Comparison

- Focusing on the Curvatures:
  - We can see that the curvatures (k_x, k_y, k_xy) are all very different in their values
    - This suggests that the bending stiffness is different in each laminate due to the orientation-dependent bending stiffness coefficients
  - A laminate with lower stiffness will exhibit lower curvature in that direction
    - In comparison between both cases and taking all curvatures into consideration, we can see that the case (a) :
      - Has ± 45 plies, which shows that its bending stiffness in both x and y directions to be reduced, leading to higher curvatures
      - Although k_xy has a significantly higher curvature, this simply indicated shear coupling, which is expected with ± 45 plies
    - In comparison with case (b):
      - The k_xy is lower than in case (a), meaning that ± 30 plies provide better torsional stiffness compared to ± 45 plies
      - Curvatures in k_y direction are similar
      - Reduction of k_x is due to the ± 30 plies increasing the axial stiffness of the composite